

February 2016
M. Sc. Ist Semester Examination

PHYSICS
Paper I : Mathematical Physics

Time 3 Hours]

[Max. Marks : Regular 85 / Private 100

Note : This question paper is meant for all Regular and Private students. Answer all five questions. All questions carry equal marks. The blind candidates will be given 60 minutes extra time.

- 1 (a) For the Bessel function of first kind or order n prove that :

$$e^{(x/2)\left(t - \frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n.$$

- (b) Prove the recursion relation :

$$(i) \quad 2n J_n(x) = x [J_{n+1}(x) + J_{n-1}(x)]$$

$$(ii) \quad J_0'(x) = -J_1(x).$$

$$(iii) \quad \frac{d}{dx} [x^n J_n(x)] = x^n J_{n-1}(x).$$

$$(iv) \quad \frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$$

OR

Prove that Hermite Polynomial is the solution of the differential equation :
 $y'' - 2xy' + 2xy = 0$.

2. Evaluate :

$$(a) \quad L^{-1} \left\{ \frac{3s-2}{s^3(s^2+4)} \right\}$$

$$(b) \quad \text{Find } F(t) \text{ in its Laplace transform is } \frac{8}{(s^2+4)^2} \text{ by using result } L(t \sin 2t) = \frac{2s}{(s^2+4)^2}.$$

- (c) Prove that (using Laplace transform) :

$$\int_0^{\infty} \frac{\sin(xt)}{\sqrt{x}} dx = \frac{\sqrt{2}\pi}{2s^{1/2}}.$$

OR

- 5 (a) State and prove Convolution Theorem. Show the uses of this theorem with the help of examples

- (b) Show that the phase angle is involved in Fourier Series. Find the Fourier Sine transform of $[F(x) = x]$ such that $0 < x < 2$.

3. Solve the Inhomogeneous Differential Equation

$$\frac{d^2 \psi}{dx^2} = f(x).$$

- (i) Subject to the homogeneous boundary condition :

$$\psi(0) = \psi(1) = 0.$$

- (ii) Subject to inhomogeneous boundary condition :

$$\psi(0) = \alpha_1 \text{ and } \psi(1) = \alpha_2.$$

OR

- (a) Determine the Green's function in terms of eigen values and eigen function of operator $\frac{d^2}{dx^2}$ for the differential equation :

$$\frac{d^2 \psi}{dx^2} = f(x) \quad 0 \leq x \leq 1$$

with boundary conditions

$$\psi(0) = 0 = \psi(1)$$

- (b) Construct the Green's function for the Legendre differential equation :

$$\frac{d}{dx} \left[(1-x^2) \frac{d\psi}{dx} \right] - \lambda \psi = f(x) \quad -1 \leq x \leq +1$$

for the simple case corresponding to $\lambda = 0$ with the boundary conditions that $\psi(x)$ is finite at $x = \pm 1$. Also obtain the bilinear form of the Green's function for the above eq in the general case under the same boundary conditions.

- (a) Using Cauchy's integral formula, evaluate the integral :

$$\oint \frac{z^2}{(z^2 - 1)} dz$$

around the unit circle with centre at (i) $z = 1$, (ii) $z = -1$ and (iii) $z = 1/2$.

- (b) Obtain the Laurent Series Expansion of $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region :

- (i) $1 < |z| < 2$.
 (ii) Exterior to the circle $|z| = 2$.
 (iii) $0 < |z - 1| < 1$.

OR

- (a) State and prove Cauchy's Residue Theorem.

- (b) Prove that :

$$I = \int_0^{2\pi} \frac{d\theta}{2 + \cos \theta} = \frac{2\pi}{\sqrt{3}}$$

5. Solve any four of the following :

- (a) Starting with the Legendre differential equation for $P_m(x)$ derive the differential equation for the associated Legendre Polynomials $P_m^n(x)$.

- (b) Obtain Recurrence formulae for Hermite Polynomials.

- (c) Explain Piecewise Continuity and obtain the condition for the existence of Laplace transforms.

- (d) Complete the table using Laplace transforms :

$F(t)$	$f(s) = L \{ F(t) \}$
$\sin kt$
e^{at}
t^n

- (e) Write the properties of Green's function.

- (f) Write the Green's function for ∇^2 operator.

- (g) Write Cauchy-Riemann Conditions.

- (h) $\int_0^\infty \frac{du}{1+u^2} = \frac{\pi}{2}$.