Roll No. ....
800 -/6/15/20

DS-218-N

January 2024

M. A. / M. Sc. I Semester Examination

MATHEMATICS

PAPER IV : COMPLEX ANALYSIS – I

Time 3 Hours] [Max. Marks : Regular 85 / Private 100
[Min. Marks : Regular 28 / Private 33

Note : This question paper is meant for all Regular and Private students. Answer all five questions. All questions carry equal marks. The blind candidates will be given 60 minutes extra time.

1. Answer any two parts :
   (a) Define Modulus and Argument of a Complex Number and find the Modulus and Argument of the following Complex Numbers :
   (i) $\frac{1-i}{1+i}$ (ii) $\frac{1}{2}\left(\frac{2-i}{3+i}+\frac{2+i}{3-i}\right)$.
   (b) State and prove necessary condition for a function $f(z)$ to be Analytic.
   (c) State and prove Cauchy's Inequality of the Complex Numbers.

2. State and prove Cauchy-Gaursat Theorem.

OR

Let $f(z)$ be analytic within and on closed contour C and let a be any point within C. Then prove that derivatives of all orders are analytic and given by :

$$f^{(n)}(a) = \frac{n!}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}}\, dz.$$

3. Answer any two parts :
   (a) State and prove Morera's Theorem.
   (b) State and prove Fundamental Theorem of Algebra.
   (c) State and prove Taylor's Theorem.

4. Answer any two parts :
   (a) State and prove Maximum Modulus Principle.
   (b) Show that $\sin\left\{c\left(z+\frac{1}{z}\right)\right\}$ can be expanded in a series of the type :

$$\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^{-n}$$

in which the coefficients of both z^n and z^{-n} are $\frac{1}{2\pi}\int_0^{2\pi} \sin(2c\cos\theta)\cos n\theta\, d\theta$.

   (c) Suppose that we have obtained in any manner or as the definition of $f(z)$ the formula :

$$f(z) = \sum_{m=-\infty}^{\infty} P_m (z-a)^m \qquad \{R_1 < |z-a| < R_2\}$$

where R_1 and R_2 are radii of concentric circles C_1 and C_2 with centre a then prove that the series is necessarily identical with the Laurent's Series for $f(z)$ in the specified annulus.

5. Answer any two parts

(a) Prove that the set of all bilinear transformation forms a non-abelian group under the Product of Transformations.

(b) Find the fixed points and the normal form of the bilinear transformation :

(i) $w = \frac{z - 1}{z + 1}$ (ii) $w = \frac{3iz + 1}{z + i}$.

(c) If $w = f(z)$ represents a conformal mapping of a domain D in the z-plane into a domain D′ of the w-plane then prove that $f(z)$ is an analytic function of z in D