Roll No. ....
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DS-218-N

February 2022

M. A. / M. Sc. I Semester Examination

MATHEMATICS

Paper IV : Complex Analysis – 1

Time 3 Hours] [Max. Marks : Regular 85 / Private 100
[Min. Marks : Regular 28 / Private 33

Note : This question paper is meant for all Regular and Private students. Answer all five questions. All questions carry equal marks. The blind candidates will be given 60 minutes extra time. Attempt any two parts from each question.

1. (a) State and prove necessary condition for a complex valued function $f(z)$ to be analytic.

 (b) (i) State and prove Cauchy's Inequality.

 (ii) Define Harmonic function and prove that Real and Imaginary parts of an analytic function satisfy Laplace's equation.

 (c) (i) If z_1 and z_2 be two complex numbers than prove that :

$$\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$$

 (ii) If $f: G \to C$ is differentiable at a point z_0 then f is continuous at z_0.

2. (a) State and prove Cauchy-Gaursat Theorem.

 (b) Evaluate $\int_C \frac{z^2-z+1}{z-1}\, dz$, C is the circle :

 (i) $|z| = 1$ (ii) $|z| = 1/2$.

 (c) (i) State and prove Cauchy's Fundamental Theorem.

 (ii) Evaluate :

$$\int_0^{2+i} z^2\, dz$$

3. (a) State and prove Morera's Theorem.

 (b) Obtain the expression for :

$$f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$$

 where $1 < |z| < 4$.

 (c) (i) Prove that a bounded entire function is constant.

 (ii) State and prove the Fundamental Theorem of Algebra.

4. (a) State and prove the Maximum Modulus Principle.

 (b) Obtain Laurent Series which represents the function :

$$f(z) = \frac{z^2-1}{(z+2)(z+3)}$$

 in the regions :

 (i) $2 < |z| < 3$ (ii) $|z| > 3$.

 (c) Prove that the function $\sin\left[c\left(z + {}^1/_2\right)\right]$ can be expanded in a series of the type

$$\sum_{n=0}^{\infty} a_n z^n + \sum_{n=1}^{\infty} b_n z^n$$

 in which the coefficients of both z^n and z^{-n} are

$$\frac{1}{2\pi}\int_0^{2\pi} \sin(2c\cos\theta)\cos n\theta\, d\theta$$

P. T. O.

5. (a) (i) Define Bilinear Transformation. Prove that product of two Bilinear Transformation is a Bilinear Transformation.

(ii) Find the Mobius Transformation which maps $0, 1, \infty$ into $1, i$ and -1 respectively.

(b) Define Cross Ratio. Prove that every bilinear transformation maps circle or straight lines into circles or straight lines and inverse points into inverse.

(c) State and prove necessary condition for conformal mapping.
