Roll No. ....

500 -/-/15/20

SF-438-A

March – April 2022

Bachelor of Computer Applications (BCA) Examination

Fifth Semester

BCA-501 : LINEAR ALGEBRA AND GEOMETRY

Time 3 Hours] [Max. Marks 40
[Min. Marks 13

Note : Attempt any two sub-parts of a questions. All questions carry equal marks.

1. Solve any two parts :
   (a) If G be the set of the non-zero real numbers and let $a * b = \frac{ab}{3}$ then show that (G, *) is an abelian group.
   (b) State and prove Lagrange's Theorem.
   (c) The homomorphism $f : G \to G'$ is an isomorphism if and only if Kerf = $\{e\}$.

2. Solve any two parts :
   (a) The necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a vector subspace of V is $a, b \in f$ and $\alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.
   (b) Show that the vectors (2, 1, 4), (1, −1, 2), (3, 1, −2) form a basis for R^3.
   (c) Show that the mapping $f : V_3(R) \to V_2(R)$ defined by $f(a, b, c) = (c,\ a + b)$ is a linear transformation.

3. Solve any two parts :
   (a) Show that the mapping $T : V_2(R) \to V_3(R)$ defined by $T : (a, b) = (a + b, a - b, b)$ is a linear transformation from $V_2(R)$ into $V_3(R)$. Find the range, rank, nullspace and nullity of T.
   (b) Find the matrix of the following linear maps with respect to the standard basis of R^3. $T : R^3 \to R^3$ defined by $T(x, y, z) = (z, y + z, x + y + z)$.
   (c) Find the characteristic roots and corresponding characteristic vectors of the matrix :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

4. Solve any two parts :
   (a) Show that the point $(5, -2, 3)$ lies on the Paraboloid $2x^2 - 5y^2 = 10z$. Find the tangent plane and the normal line at this point.
   (b) Find the equations of the tangent planes to the ellipsoid $7x^2 + 5y^2 + 3z^2 = 60$ which pass through the line $7x + 10y = 30, 5y - 3z = 0$.
   (c) Find the equation of the normal to a ellipsoid at the point (α, β, γ).

5. Solve any two parts :
   (a) Prove the condition that the plane $ax + by + cz = 0$ may cut the cone $xy + yz + zx = 0$ in perpendicular lines is $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.
   (b) Find the equation of the right circular cone whose axis $x = y = z$. Vertex is origin and whose semi-vertical angle is 45°.
   (c) Find the equation to the cylinder whose generators are parallel to the line $x = \frac{y}{-2} = \frac{z}{3}$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$.
