March 2011

Bachelor of Computer Application (BCA) Examination

V Semester

Discrete Mathematics and Linear Algebra

Time 3 Hours] [Max. Marks 40

Note : Solve any two parts from each question. All questions carry equal marks.

1. (a) Show that $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ is a tautology.
   (b) Prove that the distributive law $x(y + z) = xy + xz$ is valid.
   (c) For any two elements a and b of Boolean Algebra B prove :
   (i) $(a + b)' = a'.b'$ (ii) $(a + b)' = a' + b' \; \forall \; a, b \in B$.
2. (a) Find the disjunctive normal form for the function $F(x, y, z) = (x + y).z$.
   (b) Write the following functions into conjunctive normal form :
   (i) $F(x, y, z) = (x + y + z)(xy + x'z)'$ (ii) $f(x, y, z) = (x + y)(x + y')(x' + z)$.
   (c) Explain any two of the following terms;
   (i) Binary Search Tree (ii) Binomial Network
   (iii) Many Terminal Networks.
3. (a) State ad prove Lagrange's theorem.
   (b) Show that $A \times B \neq B \times A$, when A and B are non empty, unless $A = B$.
   (c) Reduce the following matrix to its normal form and find its rank and nullity :

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

4. (a) Show that the subset {(3, 4, -1), (1,2,0), (1, 2, 0), (1, 0, -1)} of R^3 is linearly dependent.
   (b) If R_1 = [3 1 -4], R_2 = [2 2 -3] and R_3 = [0 -4 1] show that the row matrices R_1 and R_2 are liearly independent.
   (c) If $F : R^4 \rightarrow R^3$ be the linear mapping defined by :
   $$F(x, y, z, t) = (x - y + z + t,\ x + 2z - t,\ x + y + 3z - 3t)$$

   find a basis and dimension of :
   (i) the image of F (ii) the Kernel of F.
5. (a) Find all the given values and the corresponding given vectors of the matrix :

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

   (b) Find the characteristics equation of the matrix :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

   and verify that it is satisfied by A and hence obtain A^{-1}.
   (c) Show that the matrix :

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$$

   satisfies Cayley-Hamilton theorem.

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