

Bachelor of Computer Application (BCA) Examination  
V Semester

**Discrete Mathematics and Linear Algebra**

Time : 3 Hours ]

[ Max. Marks : 40

**Note : Solve any two parts from each question. All question carry equal marks.**

1. (a) Define logical equivalence and show that:  
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r).$$
  
(b) State and prove Demorgan's Laws for Boolean Algebra.  
(c) Draw the switching circuit for the switching function  $F(x, y, z) = x \cdot y \cdot z + (x + y) \cdot (x + z)$  and replace it by a simple one.
2. (a) Write the function  $(xy' + xz)'$  in conjunctive normal form.  
(b) Write the function :  $f(x, y, z) = (x + y + z)(xy + x'z')$  into Disjunctive normal form.  
(c) Define Binomial Net and draw the Binomial Net for following flow function:  
$$F(x, y, z) = xyz + xyz' + x'yz + xy'z$$
3. (a) Prove that:  
$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$
  
(b) Show that a necessary and sufficient condition for a non empty subset  $H$  of a group  $G$  to be a subgroup is that:  
$$a \in H, b \in H \Rightarrow ab^{-1} \in H$$
  
where  $b^{-1}$  is the inverse of  $b$  in  $G$ .  
(c) State and prove Lagrange's theorem.
4. (a) Prove that, the set of all ordered  $n$  types of the elements of  $F$  with vector addition and scalar multiplication is a vector space over  $F$ , where  $F$  be an arbitrary field.  
(b) Show that the union of two subspace is subspace if and only if one is contained in other.  
(c) Show that the Kernel of a linear map is a subspace of  $U(F)$ .

5. (a) State and prove Cayley-Hamilton theorem.  
(b) Find the eigen values and eigen vectors of matrix :

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

- (c) Find the rank of matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

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