February 2014

Bachelor of Computer Applications (BCA) Examination

V Semester

Discrete Mathematics and Linear Algebra

Time 3 Hours] **[Max. Marks 40**

Note : Solve any two parts from each question. All questions carry equal marks.

1. (a) Define Universal and Existential Quantifiers. Give examples of each. Explain Negation of Quantifiers.

   (b) Verify the following relations using truth tables :

   (i) $(p \to q) \equiv (\sim p \vee q)$ (ii) $(p \to (q \to r)) \equiv ((p \wedge q) \to r)$.

   (c) Draw a circuit diagram for following Boolean function and replace it by simpler one:

   $$F(x, y, z) = [(x + y).(z + y')] + y.(x' + z')\}.$$

2. (a) Find sum-of-products expansion for the function $F(x, y, z) = (x+y).z'$.

   (b) Construct circuits that produce the following output $(x+y).x'$.

   (c) Explain any two of the following terms with the help of example :

   (i) Binary tree (ii) Spanning tree (iii) Binary search tree.

   (a) State and prove Lagrange's theorem.

   (b) Define a normal sub group of a group. Give an example. Justify your answer.

   (c) Let Z be a ring of integers and let p be a prime number. Define a mapping $f : Z \to Z^*$ such that $f(n) = np$ for all $n \in Z$. Show that f is homomorphism. Find kernel of f.

4. (a) Prove that the set of all ordered n-tuples over a field forms a vector space with respect to addition of n tuples and multiplication of n-tuples by an element of the field.

   (b) Prove that the intersection of any two subspaces of a vector space V(F) is also a subspace of V(F).

   (c) Show that the union of two subspaces is also a subspace if and only if one is contained in the other.

http://www.davvonline.com

5. (a) Write a matrix of a linear transformation $F: R^3 \to R^3$ where $F(x, y, z) = (x + y, y, y + z)$ with respect to standard bases. Find rank and nullity.

   (b) Find all eigen values and eigen vectors of the matrix :

   $$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

   (c) State and prove Caley-Hamilton theorem.

□□□

http://www.davvonline.com