December 2007

Bachelor of Computer Application (BCA) Examination

V Semester

Discrete Mathematics and Linear Algebra

Time : 3 Hours] [Max. Marks : 40

Note : Solve any two parts from each question. All questions carry equal marks.

1. (a) Define Universal and Existential Quantifiers. Give examples of each. Explain Negation of Quantifiers.

   (b) Verify following relations using truth tables :

   (i) $(p \rightarrow q) = (\sim p \vee q)$ (ii) $(p \rightarrow (Q \rightarrow R)) = (((p \wedge Q) \rightarrow R)$.

   (c) Draw a circuit for following Boolean function and replace it by simpler one :

   $F(x, y, z) = [(x + y) - (z + y)] + [y . (x' + z')]$.

2. (a) Obtain conjunctive normal form of $- (p \vee q) \Leftrightarrow (p \wedge Q)$.

   (b) Obtain disjunctive normal form of $- (p \vee q) \Leftrightarrow (p \wedge Q)$

   (c) State and prove Bools expansion theorem.

3. (a) State and prove Lagrange's theorem.

   (b) Define a normal sub group of a group. Give an example- Justify your answer.

   (c) Let Z. be a ring of integers and let p be a prime number. Define a mapping $f : Z \rightarrow Z$ such that $f(n) = np$ for all $n \in Z$. Show that f_{is} homomorphis in. Find kernel of f.

4. (a) Define a set of linearly independent vectors in a vector space. Whether (1, 1, 2), (1, 3, 0) and (2, 0, 4) are linearly independent in R^3 ?

   (b) Find the range and the kernel of the linear transformation $f : R^3 \rightarrow R^3$ defined as $f(x, y, z) = (x + y, y, y + z)$ First, show that it is a linear transformation.

   (c) Define range and kernel of a linear transformation. Show that a range is a subspace of co-domain.

5. (a) Write a matrix of a linear transformation given in Q. 4(b) with respect to standard bases Find rank and nullity.

   (b) Find all eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

   (c) State and prove Caley-Hamilton theorem.

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