May 2009

Bachelor of Computer Application (BCA) Examination

IV Semester (Autonomous)

Coordinate Geometry of three Dimensions

Time : 3 Hours] [Max. Marks : 40

Note- All questions are compulsory and carry equal marks. Solve any two parts from each question.

1. (a) Find the equation of a straight line which passes through the point (2, -6, 5) and which is perpendicular to the plane containing the points (2, -3, -4), (-3, 2, 3.5) and (2.5, 1, -1). Also find the direction cosines of the straight line.

   (b) Find the length and Equation of the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4};\ \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

   (c) Prove that the four planes $my + nz = 0$; $nz + lx = 0$; $lx + my = 0$; $(x + my) + nz = p$ form a tetrahedron whose volume is $2p^3/3lmn$.

2. (a) Prove that the circles $x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$, $5y + 6z + 1 = 0$; And $x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0$, $x + 2y - 7z =$ lie on the same sphere, and find it's equation.

   (b) Find the principal planes of the conicoid

$$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy + 2z = 0$$

   (c) If the spheres $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$ and $x^2 + y^2 + z^2 - 6y - 6z + 6 = 0$ are the members of a coaxial system of the spheres find the limiting points of the system.

3. (a) Prove that the Locus of the point of intersection of three tangent planes to the parabolid $x^2/a + y^2/b = 2z$ which are at right angles is the plane $Z + 1/2 (a + b) = 0$.

   (b) Find the Equation of the polar plane of the point (-1, 2, 3) with respect to the conicoid.

$$3x^2 + 4y^2 - z^2 - yz + 2zx + 3xy - 4x + 5y + 7z - 10 = 0$$

   (c) Find the condition that the plane $lx + my + nz = p$, touches $x^2/a + y^2/b = 2z$.

4. (a) Prove that the six normal may be drawn to the Ellipsoid from a given point (α, β, γ).

   (b) Find the Equation of the tangent plane to the ellipsoid $7x^2 + 5y^2 + 3z^2 = 60$ which passes through the straight $7x + 10y - 30 = 0$, $5y - 3z = 0$.

   (c) Define Conjugate diameters of an ellipsoid. Show that for the ellipsoid $x^2 + 4y^2 + 5z^2 = 1$ the two diameters $x/3 = y/-2 = z/4$ and $x = 0$; $2y = 5z$ are Conjugate.

5. (a) If $x/1 = y/2 = z/3$ represent one of a set of three mutually perpendicular generator of the cone $5yz - 8zx - 3xy = 0$. Find the Equation of the other two.

   (b) Find the Equation of Enveloping cylinder of the sphere $x^2 + y^2 + z^2 = a^2$ whose generating line is parallel to $x/1 = y/m = z/n$

   (c) Prove that the Equation of cone whose vertex is origin and base curve is $z = k$, $f(x, y) = 0$ is $f(xk/z, yk/z)$.

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