

June 2017

Bachelor of Computer Application (BCA) Examination
II Semester**Mathematics - II**

Time : 3 Hours]

[Max. Marks : 40

Note: All questions are compulsory and carry equal marks. Solve any two parts from each question.

1. (a) Trace the curve $y^2(2a - x) = x^3$.
(b) Trace the curve $r = 2a \cos \theta$.
(c) Test the convergence of the integral: $\int_0^{\infty} \frac{\cos mx}{x^2 + a^2} dx$.
2. (a) Prove that: $\Gamma(m) \Gamma(m + \frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$, where $m > 0$.
(b) Prove that: $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} dx = \pi$.
(c) Find the whole length of the curve: $y = \frac{1}{2}x^2 - \frac{1}{4} \log x$ from $x = 1$ to $x = 2$.
3. (a) Evaluate:
 $\int_0^3 \int_0^2 \int_0^1 (x + y + z) dx \cdot dy \cdot dz$.
(b) Using Stoke's theorem, evaluate:
 $\int_C e^x dx + 2y dy - dz$
where C is the boundary of the circle in the plane $z = 2$.
(c) If S is any closed surface enclosing a volume V and $F = xi + 2yj + 3zk$ then show that:
 $\int_S F \cdot \hat{n} dS = 6V$.
4. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that:
 $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x + y + z)^2}$.

$$(b) \text{ Let } f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Show that $f(x, y)$ is continuous but not differentiable at $(0, 0)$.

- (c) Expand $f(x, y) = x^2 + xy - y^2$ by Taylor's theorem in power of $(x - 1)$ and $(y + 2)$
5. (a) Discuss the maxima and minima of the function:

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

- (b) Test for convergence of the following series:

$$u_n = \frac{\sqrt{n}}{n^2 + 1}$$

- (c) Test the convergence of the following series:

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \dots, x > 0.$$

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