March 2011

Bachelor of Computer Application (BCA) Examination

I Semester

Mathematics–I

Time 3 Hours] [Max. Marks 40

Note : All questions are compulsory. Solve any two parts from each question. Each question carries equal marks.

1. (a) If $f(x) = \frac{x^2-3x+2}{x-2}$, find the limit of f(x) as x tends to 2.

   (b) Show that the following function is discontinuous at x = 1 :

   $f(x) = \begin{cases} x^2, \text{ when } x \neq 1 \\ 2, \text{ when } x = 1 \end{cases}$

   (c) Prove that the function f(x) = |x| is continuous at x = 0 but not differentiable at x = 0.

2. (a) Find the nth differential coefficients of $\frac{1}{1+5x+6x^2}$.

   (b) Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$.

   (c) Apply Maclaurin's theorem to prove that :

   $$\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 = \ldots$$

3. (a) Find the asymptotes of the curve $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$.

   (b) Prove that the radius of curvature at any point (x, y) of the curve

   $y = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right)$ is y^2/a.

   (c) Evaluate $\int_0^{\pi/2} x^3 \sin 3x \, dx$.

4. (a) If $\vec{r} = ae^{nt} + be^{-nt}$, where a, b are constant vectors, then prove that : $\frac{d^2\vec{r}}{dt^2} - n^2\vec{r} = 0$.

   (b) If $r^2 = x^2 + y^2 + z^2$, find grad r^n.

   (c) Find div curl f where $f = x^2y\,\vec{i} + xz\,\vec{j} + 2yz\,\vec{k}$.

5. (a) Find the inverse of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

   (b) Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

   (c) Show that the following equations are inconsistent :

   $x + y + z = -3$

   $3x + y - 2z = -2$

   $2x + 4y + 7z = 7$.

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