January 2019

Bachelor of Computer Applications (BCA) Examination

V Semester

Discrete Mathematics and Linear Algebra

Time 3 Hours] [Max.Marks 40

Note : Solve any two parts from each question. All questions carry equal marks.

1. (a) Explain ten laws of Algebra of Propositions.

(b) Construct truth table for given propositions :

(i) $p \wedge (\sim q \vee q)$

(ii) $\sim(p \vee q) \vee (\sim p \wedge \sim q)$.

(c) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology, using De-Morgan's Laws.

2. (a) Show following Boolean Expressions are equivalent to one another. Obtain their sum of products in canonical form :

(i) $(x \oplus y) * (x' \oplus z) * (y \oplus z)$

(ii) $(x * z) \oplus (x' * y) \oplus (y * 2)$

(iii) $(x \oplus y) * (x' \oplus z)$

(iv) $(x * z) \oplus (x' * y)$.

(b) Explain different theorems of Trees with examples. What are different applications of trees? Explain with examples.

(c) Convert the Boolean function :

(i) $f(x, y) = x \cdot y' + x' \cdot y + x'.y'$ into Conjunctive normal form.

(ii) Convert Boolean function $f(x, y, z) = (x' + y + z').(x' + y + z).(x + y' + z)$ in Disjunctive normal form.

3. (a) Explain ten laws of Algebra of Operations of Sets, with examples.

(b) Let $G = \{(a, b) \mid a, b \in R, a \neq 0)$. We define a binary operation $*$ on G by $(a, b) * (c, d) = (ac, bc + d)$ for all $(a, b), (c, d) \in G$. Show that $(G, *)$ is a Group.

(c) (i) Let 'G' be a group. For fixed element of 'G', Let $G_x = (a \in G : ax = xa\}$. Show that G_x is a sub-group of G for all $x \in G$.

(ii) If 'G' is a abelian group with identity 'e'. then prove that all elements of x of G, satisfying the equation, $x^2 = e$, forms a subgroup H of G.

4. (a) Define Vector Space with example. Prove that vector space of all ordered n-tuples over the field 'F' of real numbers by closure, associative, commutative and additive identity and additive inverse in vector space V.

where F = Field of real numbers

V = Set of ordered n-tuples of real numbers

$V = ((a_1, a_2, a_3 \ldots\ldots a_n) : a_1, a_2, a_3 \in F)$

and $\alpha = (a_1, a_2, a_3 \ldots\ldots a_n)$

and $\beta = (b_1, b_2, b_3 \ldots\ldots b_n)$.

(b Show that vector's :

(i) (1, 2, 0), (0, 3, 0), (-1, 0, 1) are linearly dependent in vector space 'V_3' of real number R.

(ii) $S = \{(1, 0, 1), (1, 1, 0), (-1, 0, -1)$ is linearly dependent in V_3 (R).

(c) (i) Let $T : V_2 (R) \to V_3 (R)$ defined by $T(x, y) = (x - y, y, x + y)$. Find range, null space, rank, nullity and verify rank-nullity theorem.

(ii) Show that linear transformation $T : R^3 \to R^3$ defined by :

$T(x, y, z) = T(x + z, x + y + 2z, x - y)$ is an isomorphism.

5. (a) (i) State and prove Cayley Hamilton Theorem using suitable mathematical expression.

(ii) Define eigen values and eigen vectors. How they are determined ? Explain with example,

(b) Find eigen values and eigen vectors of the matrix = $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

(c) State and prove the Fundamental Theorem of Vector Space Homomorphism (with examples and mathematical expressions).

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