December 2008

Bachelor of Computer Application (BCA) Examination

**V Semester**

# Discrete Mathematics and Linear Algebra

Time : 3 Hours ] [ Max. Marks : 40

**Note : Solve any two parts from each questions. All questions carry equal marks.**

1. (a) Define tautology and contradiction. If p and q are two statements then prove the following :
   (i) $\sim (p \wedge q) \equiv \sim p \vee \sim q$
   (ii) $\sim (p \vee q) \equiv \sim p \wedge \sim q$
   (iii) $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \Rightarrow (p \leftrightarrow r)$.
   (b) The bulb inside a car of two doors in on when one of these doors in opened and it is also on when a switch of desh–board is pressed. Draw the diagram for the control path.
   (c) In Boolean Algebra, prove that :
   $a . b + b . c + c . a = (a + b)(b + c)(c + a)$.
2. (a) Prove that the number of minimal Boolean function in n–variables are $2^n$.
   (b) Prove that : If in complete disjunctive normal form or complete canonical form in `n' variables, each variables is assigned arbitrarily the value 0 or 1, then just one term will have the value 1 while others will have the value 0.
   (c) Explain the following by giving examples :
   (i) Many Termial Network (ii) Biominal Net
3. (a) Show that the mapping $f : R \to R$, $f(x) = \frac{1}{x}$, $x \neq 0$ and $x \in R$ is one–one onto, where R is the set of non–zero real numbers.
   (b) If $f : X \to Y$ is one–one and onto then prove that $f^{-1} : Y \to X$ is also one–one and onto.
   (c) Let G be a cyclic group then prove :
   (i) If G is of infinite order, then G is isomorphic to (Z, +).
   (ii) If G is of finite order with (G) = n then G is isomorphic to $(Z_n, +)$.



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4. (a) Define the kernel of a linear map. If f : U(f) F – V(F) be a linear map then show that kernel of f is a subspace of U(f).
   (b) Prove that the functions $x^2 - 2x$, $2x^2 + x$, $x^2$ on $(-\infty, \infty)$ are the solutions of differential equation :
   $$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$
   Determine whether they are linearly independent and whether they form a basis for solution space.
   (c) Prove that the sets of all vectors in a plane is vector space over the field of real numbers.
5. (a) State and prove rank–nullity theorem.
   (b) Prove that the eigen values of a unitary matrix are of unit modulus.
   (c) Find the rank of the matrix A where :
   $$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

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