June 2017

Bachelor of Computer Application (BCA) Examination

IV Semester

Coordinate Geometry of three Dimensions

Time 3 Hours] [Max. Marks 40

Note : All questions are compulsory and carry equal marks. Solve any two parts from each question.

1. (a) Show that the equations of lines given by

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4} \text{ and } \frac{x-3}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

are Coplanar.

(b) Prove that the lines :

$$\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7} \text{ and } \frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$$

intersect and find the coordinate of the point of intersection.

(c) Prove that the angle between two diagonals of a cube is

$$\cos^{-1}\left(\frac{1}{3}\right)$$

2. (a) If the sphere $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$ and $x^2 + y^2 + z^2 - 6y + 6z + 6 = 0$ are member of coaxial system of sphere. Find the limiting points of the system.

(b) Find the coordinate of the centre (if any) of the quadratic surface :

$$z^2 - yz + zx + xy - 2y + 2z + 2 = 0$$

(c) Find the equation of diametral plane of the conicoid, corresponding to the system of chord parallel to the straight line :

$$\frac{x-1}{4}=\frac{y+2}{3}=\frac{z}{-2}$$

3. (a) Find the equation of polar plane of the point (-1, 2, 3) with respect to the conicoid :

$$3x^2 + 4y^2 - z^2 - yz + 2zx + 3xy - 4x + 5y + 7z - 10 = 0$$

(b) Find the equation of tangent plane to paraboloid $ax^2 + by^2 = 2cz$ at the point (α, β, γ)

(c) Find the locus of chord of the conicoid :

$$x^2 + 2y^2 - z^2 + 4yz - zx + x + 2y - 1 = 0$$

which are bisected at the point (1, 0, 4).

4. (a) Show that the plane $3x + 12y - 6z + 17 = 0$ touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$ and find the point of contact.

(b) Prove that six normals can be drawn to an ellipsoid from a given point (x_1, y_1, z_1).

(c) A tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ touches the coordinate axes in the points P, Q and R. Prove that the centroid of triangle PQR lie on the surface $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9$.

5. (a) Prove that the equation :

$4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$

represent a cone. Find the coordinate of its vertex.

(b) Find the reciprocal cone of the cone :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$

(c) Find the equation of enveloping cylinder of the sphere :

$x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the line $x = y = z$.

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