July 2014

Bachelor of Computer Application (BCA) Examination

IV Semester

Coordinate Geometry of three Dimensions

Time : 3 Hours] [Max. Marks : 40

Note : All questions are compulsory and carry equal marks. Solve any two parts from each questions.

1. (a) Prove that the acute angle between the lines whose direction cosines are given by the relation $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is $\pi/3$.

   (b) Find the length and equations of shortest distance between the straight lines :

   $$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$

   (c) Show that the lines whose equations are given by :

   $$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4}, \frac{x-3}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

   are coplanar.

2. (a) Find the equation of the sphere with centre at (2, 3, –4) and touching the plane :

   $$2x + 6y - 3z + 15 = 0.$$

   (b) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.

   (c) Find the principal planes of the conicoid $3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy + 2z = 0$.

3. (a) Find the locus of the chord of the conicoid $x^2 + 2y^2 - 3z^2 - 4yz + 8zx - 12xy + 1 = 0$ which bisected at the point (1, –1, 3).

   (b) Find the equations of the plane which cuts the paraboloid $x^2 - 2y^2 = z$ in a conic with its centre at (2, 3/2, 4).

   (c) Prove that the normals from (α, β, γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lies on the cone $\frac{\alpha}{x-\alpha}-\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-y}=0$.

4. (a) Find the condition that the plane $lx + my + nz = p$ may touch the conicoid $ax^2 + by^2 + cz^2 = 1$.

(b) Find the centre of the conic :

$$\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4}=1;\ 2x+2y-z=3.$$

(c) Prove that six normals can be drawn to an ellipsoid from a given point (x_1, y_1, z_1).

5. (a) Find the equation of cone whose vertex is (0, 0, 3) and base curve is $x^2 + y^2 = 4$, $z = 0$.

(b) Find the equation of enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y - 2 = 0$ with the vertex at (1, 1, 1).

(c) Find the equation of quadric cylinder which intersect the curve $ax^2 + by^2 + cz^2 = 1$; $lx + my + nz = p$ and whose generator are parallel to the axis of x.

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