June-July 2007

Bachelor of Computer Application (BCA) Examination

IV Semester

Coordinate Geometry of Three Dimensions

Time : 3 Hours] [Max. Marks : 50

Note : All questions are compulsory and carry equal marks. Solve any two parts from each question.

1. (a) Find the cylindrical & spherical equations of locus of a point if its rectangular cartesian equation is $x^2 + y^2 = z^2 \tan^2\alpha$, where a is constant.

   (b) Show that the straight lines

   $$\frac{x+3}{2} = \frac{y+5}{3} = \frac{z-7}{-3}; \frac{x+1}{4} = \frac{y+1}{5} = \frac{z+1}{-1}$$

   are coplanar and find the equation of the plane containing them.

   (c) Find the area of the triangle formed by joining the points (2, 1, 1), (3, 1, 2) and (-4, 0, 1)

2. (a) Find the equation to the sphere which passes through the pionts (4, 1, 0), (2, -3, 4) and (1, 0, 0) and touches the plane $2x + 2y - z = 1$

   (b) If the spheres $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$ and $x^2 + y^2 + z^2 - 6y - 6z + 6 = 0$ are members of a coaxial system of spheres. Find the limiting point of the system.

   (c) Find the principal direction and principal planes of the conicoid

   $8x^2 + 7y^2 + 3z^2 - 8yz + 4xz - 12xy + 2x - 8y + 1 = 0$

3. (a) Find the locus of thye chord of the conicoid $x^2 + 2y^2 - z^2 + 4yz - zx + x + 2y - 1 = 0$ which bisected at the point (1, 0, 4).

   (b) Find the equation of the polar plane of the point (-1, 2, 3) with respect to concicoid :

   $3x^2 + 4y^2 - z^2 - yz + 2xz + 3xy - 4x + 5y + 7z - 10 = 0$

   (c) Prove that the equation of tangent plane at the pont (x_1, y_1, z_1) to the paraboloid :

   $$\frac{x^2}{a} + \frac{y^2}{b} = 2z \text{ is } \frac{xx_2}{a} + \frac{yy_1}{b} = (z + z_1)$$

4. (a) Prove that six normal can be drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ from a point (α, β, γ)

(b) Find the equation of the tangent plane to the ellipsoid $7x^2 + 5y^2 + 3z^2 = 60$ which passes through the straight line $7x + 10y - 30 = 0$; $5y - 3z = 0$.

(c) Find the centre of the central conic definedby the equations

$$\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{4} = 1;\ 2x + 2y - z = 3.$$

5. (a) Findthe equation of the enveloping coneof the sphere $x^2 + y^2 + z^2 = a^2$ with thevertex at the point (x_1, y_1, z_1)

(b) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the co-ordinates axes in A, B, C prove that the equation of the cone generated by the lines drawnfrom 0 to meet the circle ABC is :

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=D.$$

(c) Find the equation of the right circular cylinder of radius z whose ax is is the line $\frac{x-1}{2}=\frac{y}{3}=\frac{z-3}{1}$.

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