May 2003

Bachelor of Computer Application (BCA) Examination

II Semester

Statistical Methods

Time 3 Hours] [Max. Marks 40

Note : *Attempt all five questions. Each question carries equal marks.*

1. (a) For the distribution

$$F(x) = \frac{1}{\theta^p \Gamma(P)} \exp(-x/\theta) x^{p-1}, \; 0 \leq x < \infty, \; p > 0, \; \theta > 0$$

Where p is known, find out the maximum likelihood estimate of θ on the basis of a random sample of size n from the distribution. Find the variance of the estimate also.

(b) Define consistency, efficiency and sufficiency on estimators with suitable example.

OR

(a) If T_1 and T_2 be two unbiased estimators of $\tau(\theta)$ with variances σ_1^2, σ_2^2 and correlation P, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination?

(b) Prove that if a sufficient estimator exists, it is a function of the maximum likelihood estimator.

2. (a) State and prove Neymann Pearson's fundamental lemma.

(b) Explain the following terms :

Best critical region, Level of significance, Composite hypothesis, Power function of a test, Powerful test.

OR

(a) The hypothesis $\mu = 50$, is rejected if mean of a sample of size 25 is either greater than 70.54 or less than 31.19. Assuming the distribution to be normal with standard deviation 50, find the level of significance and power function of the test.

(b) If $X \geq 1$, is the critical region for testing H_0 ; $\theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population :

$F(x,\theta) = \theta \exp(-\theta x), \; 0 \leq x < \infty,$

obtain type I and type II errors.

3. Let X_1 and X_2 be a random sample of size 2 from N (0, 1) and Y_1 and Y_2 be a random sample of size 2 from N (1, 1), and let Y_i's be independent of X_i's. Find the distribution of the following :

(a) $\overline{X}+\overline{Y}$

(b) $(X_1 + X_2)^2 / (X_2 - X_1)^2$

(c) $(Y_1 + Y_2 - 2)^2 / (X_2 - X_1)^2$

(d) $(X_1 + X_2)/\sqrt{[(X_2 - X_1)^2 + (Y_2 - Y_1)^2]/2}$

(e) $[(Y_1 - Y_2)^2 + (X_1 - X_2)^2 + (X_1 + X_2)^2]/2.$

OR

(a) Determine mode and point of inflexion of F-distribution.

(b) Determine mode and skewness of Chi-square distribution.

4. Write note on any two of the following :

Sign test, median test, Wilcoxon's run test, Contingency table.

5. Discuss two-way classification of analysis of variance with one observation per cell.

OR

(a) Explain the basic principles of design of experiments.

(b) Write a note on completely randomized design.

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