January 2016

Bachelor of Computer Applications (BCA) Examination

I Semester

Mathematics - I

Time : 3 Hours] [Max. Marks : 40

Note : All questions are compulsory Solve any two parts from each question. Each question carries equal marks.

1. (a) If $f(x)=\dfrac{x^2-3x+2}{x-2}$, find the limit of f (x) as x tends to 2.

(b) Test the continuity of the following function at x = 0 :

$$F(x) = \begin{cases} x.\sin\frac{1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

(c) Show that $f(x) = x^2$ is differentiable at x = 1 and also find f' (1).

2. (a) Expand $\tan^{-1(x)}$ in power of $(x - \pi/4)$.

(b) Apply Maclaurin's theorem to prove that :

$$\log \sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + ...$$

(c) If y = A sin mx + B cos mx, then prove that $y2 + m^2y = 0$.

3. (a) Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0$.

(b) Prove that the radius of curvature at point $(a \cos^3 \theta, a \sin^3 \theta)$ of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a \sin \theta \cos \theta$.

(c) Find the equation of the tangent at point ``t" the following curve:

$$X = a (t + \sin t), Y = a (1 - \cos t).$$

4. (a) Show tat div $\hat{r} = \dfrac{2}{r}$.

(b) Prove that div (A × B) = B. curl A - A. curl B.

(c) Find the direction derivative of the function $\phi = (x^2 + y^2 + z^2)^{1/2}$ at the point (3, 12) in the direction $yz\ \hat{i} + zx\ \hat{j} + xy\ \hat{k}$.

5. (a) Find the rank and nullity of the matrix A :

$$\text{where } A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 7 \\ 3 & 3 & 0 \end{bmatrix}$$

(b) Prove that following equations are consistent and solve them:

$$x - y + z = 2$$
$$3x - y + 2z = -6$$
$$3x + y + z = -18$$

(c) Show that the following equations are inconsistent:

$$x + y + z = 3$$
$$3x + y + 2z = -2$$
$$2x + 4y + 7z = 7.$$

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