January 2015
Bachelor of Computer Applications (BCA) Examination
I Semester

(Mathematics - I)

Time : 3 Hours]   [Max. Marks : 40

Note : All questions are compulsory Solve any two parts from each question. Each question carries equal marks.

1.   (a)   Find $\lim\limits_{y \to 0} \dfrac{(x - \sin x)}{x^3}$

      (b)   Test the function $f(x) = \begin{cases} 1 + x^2 & \text{when } 0 \le x \le 1 \\ 1 - x & \text{when } x > 1 \end{cases}$ for continuity at $x = 1$

      (c)   Show that the function $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.

2.   (a)   Expand $e^{\sin x}$ by Maclaurin's theorem.

      (b)   Verify Rolle's theorem for the function $f(x) = x^3 - 12x$ in the interval $[0, 2\sqrt{3}]$

      (e)   Explain $\sin x$ in powers of $\left(x - \dfrac{\pi}{2}\right)$ by Taylor's theorem.

3.   (a)   Find the asymptotes of the curve :
        $y^3 - x^2 y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$.

      (b)   (Find the radius of curvature at a point $(\alpha \cos^3 \alpha, \alpha \sin^3 \alpha)$ of the curve $x^{2/3} + y^{2/3} = \alpha^{2/3}$.

      (e)   Evaluate $\displaystyle\int_0^{\pi/4} \tan^n x \, dx$.

4.   (a)   If $x = \cos nt \,\hat{i} + \sin nt \,\hat{j}$ where n is a constant and t varies, show that :

        $\dfrac{d^2 \vec{r}}{dt^2} = -n^2 \vec{r}$.

      (b)   If $\vec{r} = xi + yj + zk$ show that grad $r^n = nr^{n-2} \vec{r}$.

      (e)   Find the directional derivative of $\phi = xy + yz + zx$ in the direction of the vector $i + 2j + 2k$ at the point $(1, 2, 0)$.

5.   (a)   Find the rank and nullity of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 7 & 1 \\ 5 & 9 & 3 \end{bmatrix}$$

      (b)   Prove that the following equations are consistent and solve :
        $2x + 4y - z = 9$
        $3x - y + 5z = 5$
        $3x + 2y + 9z = 19$.

      (e)   If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then find A^{-1}.

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